4.0 Short-Circuit Effects

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4.1 Introduction

Like all electrical circuits, busbars need to be protected against the effects of short-circuit currents. The open construction of busbars increases the risk of faults, e.g. by the ingress of foreign bodies into air gaps, and the risk of consequent damage is high due to their high normal operating currents and the amount of energy available.

Very high currents lead to rapid and extreme overheating of the bars with consequent softening of the material and damage to the support structure. At the same time, the electromagnetic forces generated will distort the softened conductors which may break free from their supports. Resonance effects may make the situation worse.

4.2 Short-Circuit Heating of Bars

The maximum potential short-circuit current depends on the source impedance of the supply and reduces along the length of the bar as the impedance increases. For the purpose of ensuring the safety and integrity of the bar, the short circuit current should be initially calculated close to the feed end of the bar, making no allowance for bar impedance, to establish the worst case.

When a fault occurs, the short-circuit current will be many times the normal operating current and will flow until the protective device operates. Because the time duration is small – a few seconds at most – it is usual to assume adiabatic heating, in other words, that, over the time scale of interest, there is no significant cooling effect and that all the heat generated by the current flow is retained in the bar. Therefore it is assumed that the temperature rise of the bar is simply linear - this simplifies the calculation significantly and yields a conservative result.[1]

The amount of heat energy required to raise the temperature of a unit mass of material by one degree Centigrade is called the specific heat. For copper, at room temperature, the value is 385 Joule/kg/K. Knowing the mass of the bar and the energy produced by the short circuit current, the temperature rise can be calculated:

$$Q = S\,m\,t_r \text{ or}$$

$$t_r = \frac{Q}{S\,m}$$

where:

- Q is the amount of heat added to the bar (Joule)
- S is the specific heat of the bar material (J/kg/K)
- m is the mass of the bar (kg)
- t_r is the temperature rise.

The amount of energy dissipated in the bar is:

$$Q = P\,T$$

where:

- P is the power dissipated in the bar (W)
- T is the time for which the power is dissipated (seconds).

Therefore, $t_r = \frac{PT}{Sm}$ or $\Delta t_r = \frac{P}{Sm}$

where Δt_r is the rate of temperature rise in degrees C per second.

The power dissipated in the bar is given by

$$P = I^2 R \text{ or}$$

$$P = I^2 \frac{\rho l}{A}$$

[1] A similar simplification is made in determining cable behaviour under fault conditions.

where:

R is the resistance of the bar (Ω)
ρ is the resistivity of the bar material (Ωm)
l is the length of the bar (m)
A is the cross-sectional area of the bar (m^2).

Substituting for *W* and *M* gives:

$$\Delta t_r = \frac{I^2 \rho l}{A S D l A} = \frac{I^2 \rho}{A^2 SD} = \frac{\rho}{SD}\left(\frac{I}{A}\right)^2$$

where:

D is density (kg/m^3).

Two of the physical constants, specific heat and resistivity, vary considerably with temperature so it is not obvious which values should be used. Resistivity increases with temperature (by a factor of 1.6 from 20ºC to 300ºC) so increasing the energy dissipated as the temperature increases, while specific heat falls by around 8% over the same range.

Using room temperature values, and adjusting to use convenient units, gives the initial rate of temperature rise as:

$$\Delta t_r \approx 5 \times 10^3 \left(\frac{I}{A}\right)^2$$

where:

I is current in kA
A is the cross-sectional area in mm^2.

At higher temperatures – in other words, as the fault current continues to flow until the protective device operates – the rate of temperature rise will increase as the resistance of the bar increases. The worst case rate of rise at the feed end of the bar is approximately:

$$\Delta t_r \approx 8 \times 10^3 \left(\frac{I}{A}\right)^2$$

However, if the short circuit is at some distance from the feed end, the fault current magnitude will be lower due to the resistance of the bar and will reduce further as the bar temperature rises.

In practice, what is important is that the final temperature of the bar remains lower than the limiting design temperature throughout the short circuit event. The limiting temperature for copper busbars is determined by the temperature resistance of the support materials but, in any case, should not exceed ~ 200ºC. The maximum circuit breaker tripping time is $200/\Delta t_r$ seconds.

Busbars that have been subject to short circuit should be allowed to cool and inspected before being returned to service to ensure that all joints remain tight and that the mountings are secure. Note that, although the heating time – the duration of the fault – is quite short, the bar will remain at high temperature for a considerable length of time. Also, because of the very high thermal conductivity of copper, parts of the bar beyond the fault will also have become hot.

4.3 Electromagnetic Stresses

Busbars are subject to mechanical forces since each is carrying a current though the magnetic fields caused by currents in other bars. When alternating currents are flowing, the forces have a steady component, but also a vibrational component at twice the frequency of the alternating current. Under normal working conditions these forces are of little consequence.

However, if the bars are mounted on supports, each section will have a resonant frequency. If this frequency is close to twice the supply current (or any significant harmonic current), then resonant vibration of these beams may occur. This rather special and uncommon circumstance can lead to high vibrational displacements and possibly to metal fatigue or loosening of joints and connections. The problem may be avoided by choosing an appropriate spacing of the supports or cured by introducing additional intermediate supports. Methods of estimating mechanical resonant frequencies are given later.

If large currents flow, such as when a short circuit occurs, the forces can be more important. The unidirectional component of the forces, exacerbated by the vibrational component, can lead to permanent bending and distortion of the bars or damage to, and even breakage of supports.

The peak, or fully asymmetrical, short-circuit current is dependent on the power factor (cos ϕ) of the busbar system and its associated connected electrical plant. The value is obtained by multiplying the rms symmetrical current by the appropriate factor given in balanced three-phase short-circuit stresses.

The peak current, *I*, attained during the short circuit, varies with the power factor of the circuit (see Table 13):

Table 13 – Power Factor and Peak Current

Power Factor	Peak Current as Multiple of Steady State rms
0	2.828
0.07	2.55
0.20	2.2
0.25	2.1
0.30	2
0.50	1.7
0.70	1.5
1.0	1.414

The theoretical maximum for this factor is $2\sqrt{2}$ or 2.828 where cos $\phi = 0$. If the power factor of the system is not known, then a factor of 2.55 will normally be close to the actual system value, especially where generation is concerned. These peak values reduce exponentially and, after approximately 10 cycles, the factor falls to 1.0, i.e. the symmetrical rms short-circuit current. As shown in Figure 45, the peak forces therefore normally occur in the first two cycles (0.04 s).

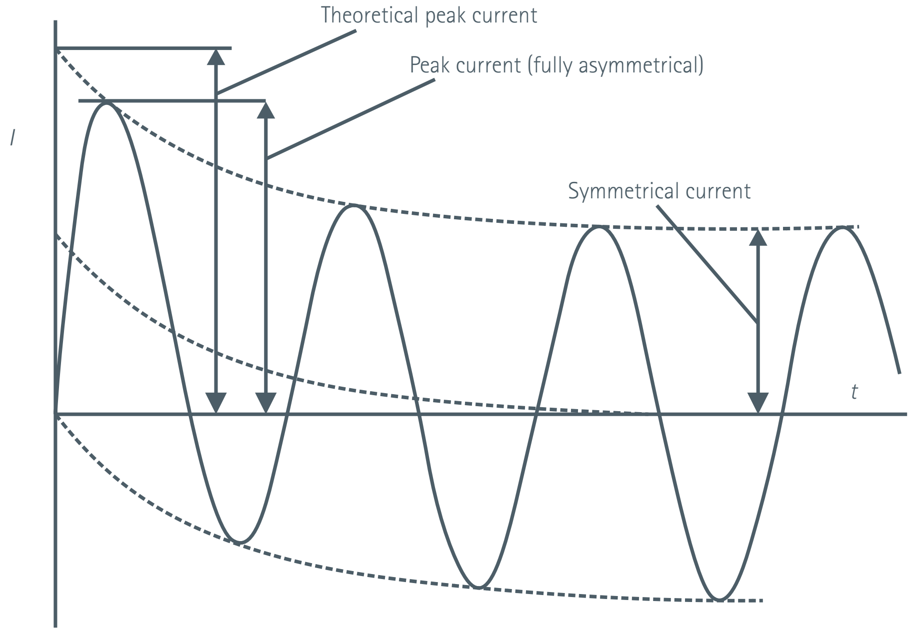


Figure 45 – Short-circuit current waveform

Bars carrying like parallel currents are attracted towards each other. Thus, if the currents in one phase are carried in separate conductors in parallel in the same phase, then the gap between them may be reduced by the mechanical forces. More usually we deal with bars carrying anti-parallel currents when the bars repel each other.

Eddy current effects in bars carrying anti-parallel currents bring the current flow in a bar closer to the sides facing the other conductor. This will lead to an increase in the repulsive force. Precise formulae for this are not available but an over-estimate of the magnitude of the force under these circumstances may be made by replacing the centre line distance in the formula for the force per unit length by the distance between the faces.

4.3.1 Estimating the Forces Between Parallel Sets of Bars

Assuming that the bars are round will give values for the forces which, used with a factor of safety in estimating strength, are normally reliable. If there are many bars in the configuration then the forces between individual pairs of bars may be simply added together, taking into account their direction. In the case of ac, one may need to take into account the phases of the currents.

Bar shape usually has a minor effect unless the bars are very close together, as will be illustrated in the example of rectangular bars.

4.3.1.1 Round Bars

For two bars with spacing *d* and carrying parallel currents I_1 and I_2, the repulsive force per unit length is

$$F = -\frac{\mu_o}{2\pi d} I_1 I_2$$

where:

- d is spacing in mm
- μ_o is absolute permeability = $4\pi \times 10^{-7}$ Henrys per metre
- F is force per unit length in N/m.

For two bars, at spacing *d*, carrying anti-parallel direct currents (i.e. go and return currents) of magnitude *I*, the repulsive force per unit length is

$$F = -\frac{\mu_o I^2}{2\pi d}$$

$$F = 0.2 \times 10^{-3} \frac{I^2}{d}$$

For two bars at spacing, *d*, carrying anti-parallel alternating currents (i.e. go and return currents) of magnitude *I*, the repulsive force per unit length is

$$F = \frac{\mu_0 I_{rms}^2}{2\pi d}[1 - cos(2\omega t)]$$

$$F = 0.2 \times 10^{-3} \frac{I_{rms}^2}{d}[1 - cos(2\omega t)]$$

where:

- d is spacing in mm
- ω = $2\pi f$
- f is frequency in Hz.

Thus we have a steady repulsive force, together with a force alternating at twice the frequency of the current of amplitude equal in magnitude to the steady component of force. The result can also be seen as a periodic force varying from zero to twice the average force.

For balanced three phase currents of rms value *I* at frequency $\omega = 2\pi f$ (ω in rad/s, f in Hz), the currents are:

$$I_A = \sqrt{2}\, I\, sin(\omega t)$$

$$I_B = \sqrt{2}\, I\, sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$I_C = \sqrt{2}\, I\, sin\left(\omega t + \frac{2\pi}{3}\right)$$

The standard triangular and inline arrays are shown in Figure 46.

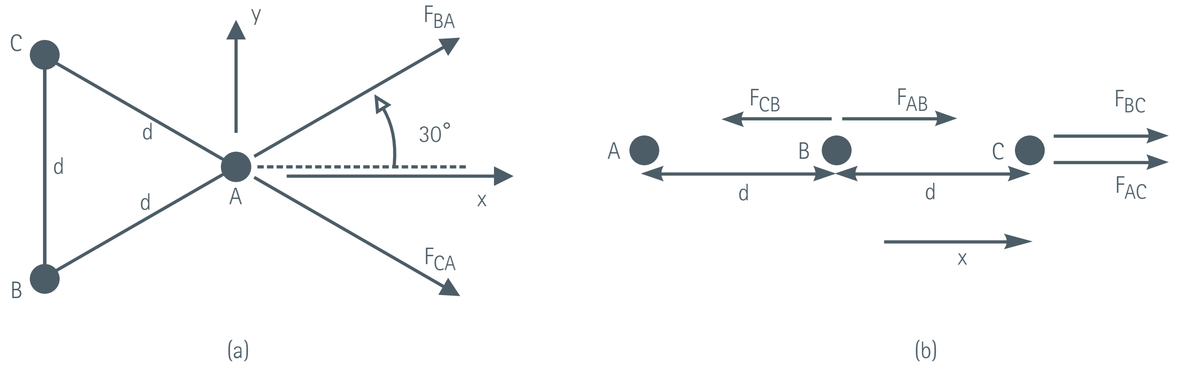


Figure 46 – Three phase system with spacings d (mm). (a) triangular array, (b) inline array

4.3.1.1.1 Triangular Array

The net force per unit length in the x-direction on bar A in Figure 46(a)

$$F_x = \frac{\mu_0 I^2}{\pi d}\frac{\sqrt{3}}{2}[1 - cos(\omega t)]$$

$$F_x \approx 0.693 \times 10^{-3}\frac{I^2}{d}[1 - cos(\omega t)]$$

The net force in the y-direction on bar A in Figure 46(a)

$$F_y = \frac{\mu_0 I^2}{\pi d}\frac{\sqrt{3}}{2} sin(2\omega t)$$

$$F_y \approx 0.693 \times 10^{-3}\frac{I^2}{d} sin(2\omega t)$$

The net result is a steady force outwards along the perpendicular to the line BC of magnitude

$$F_{x\,steady} = \frac{\mu_0 I^2}{\pi d}\frac{\sqrt{3}}{2}$$

$$F_{x\,steady} \approx 0.693 \times 10^{-3}\frac{I^2}{d}$$

The alternating forces in the x and y direction are in phase quadrature and of equal magnitude and so constitute a force tending to move the conductor in a circle. The magnitude is the same as that of the steady force.

4.3.1.1.2 In-Line Array

The force per unit length in the x-direction (i.e. an outward repulsive force) on bar C in Figure 46(b) is

$$F_C = \frac{\mu_0 I^2}{\pi d}\left[\frac{3}{8} - \frac{\sqrt{3}}{4} sin\left(2\omega t - \frac{\pi}{3}\right)\right]$$

$$F_C \approx \left[0.15 - 0.1732\ sin\left(2\omega t - \frac{\pi}{3}\right)\right] 10^{-3}\frac{I^2}{d}$$

This is a steady outward force with a superposed 2nd harmonic vibration. The amplitude of the harmonic force is slightly greater than the steady force.

The force per unit length on the centre bar, B, is purely vibrational. In the x-direction it has magnitude

$$F_B = \frac{\mu_0 I^2}{\pi d}\frac{\sqrt{3}}{2} sin\left(2\omega t - \frac{\pi}{3}\right)$$

$$F_B \approx 0.346 \times 10^{-3}\ sin\left(2\omega t - \frac{\pi}{3}\right)\frac{I^2}{d}$$

The force per unit length in the x-direction on bar A is

$$F_A = -F_B - F_C = \frac{\mu_0 I^2}{\pi d}\left[-\frac{3}{8} - \frac{\sqrt{3}}{4} sin\left(2\omega t - \frac{\pi}{3}\right)\right]$$

$$F_A \approx \left[-0.15 - 0.1732\ sin\left(2\omega t - \frac{\pi}{3}\right)\right] 10^{-3}\frac{I^2}{d}$$

This too is on average a repulsive force outwards in the negative x-direction.

4.3.1.2 Bars of Rectangular Section

For bars of rectangular section set out orderly, the force between the two may be expressed as the force computed as if they were round bars multiplied by a correction factor. Thus the force, in N/m, between two rectangular bars with centre line spacing *d*, is

$$F = K\frac{\mu_0 I^2}{2\pi d}$$

$$F = 0.2 \times 10^{-3} K\frac{I^2}{d}$$

The factor *K* depends on the length of the sides, a and b, of the rectangular cross-section and the spacing, *d*, of the centre lines.

K is shown in Figure 47 as a function of $\frac{d-a}{a+b}$

Note that the curves labelled a/b = 0 and b/a = 0 represent limiting values.

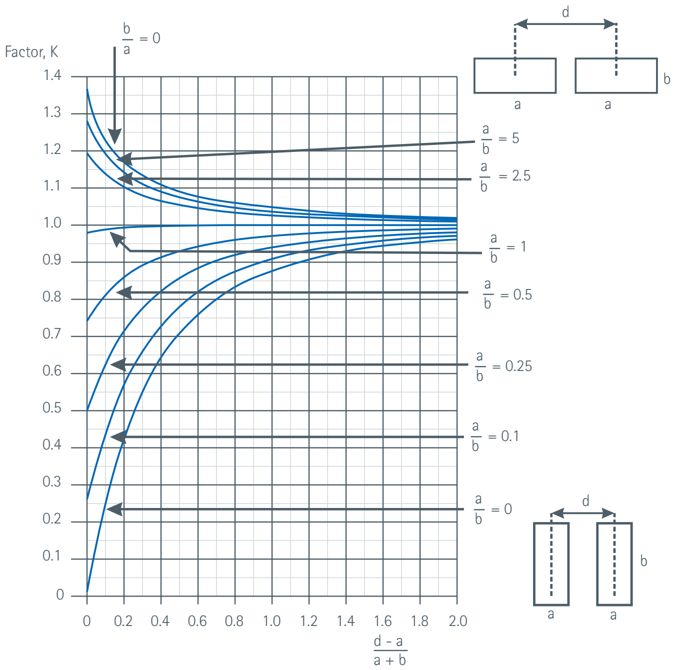


Figure 47 – Factor K for calculating the force between two bars of rectangular section

For a pair of rectangles whose long sides face each other, it will prove helpful to use the equations below in which d is replaced by b and factor K replaced by K_b which is plotted in Figure 48.

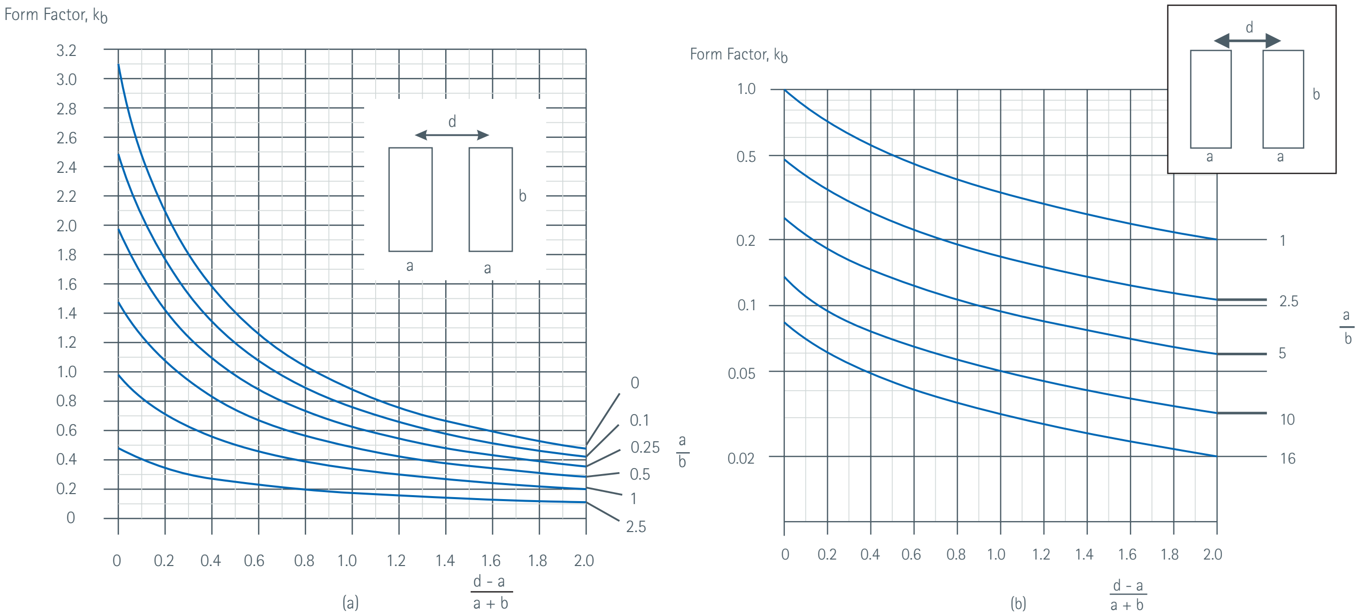


Figure 48 – Form factor K_b. (a) low values of a/b (long sides facing each other) (b) high values of a/b.

An approximate formula for K_b accurate to within 4% for $0 < \frac{d-a}{a+b} < 2$ and $\frac{1}{16} < \frac{a}{b} < 16$ is:

$$K_b \approx \frac{2.98}{\left[1 + 2.228\left(\frac{a}{b}\right)\right]\left[1 + AF + BF^2\right]}$$

where:

$$F = \frac{d-a}{a+b}$$

$$A = \frac{2.46 + 45.75\left(\frac{a}{b}\right) + 9.25\left(\frac{a}{b}\right)^2}{1 + 24.4\left(\frac{a}{b}\right) + 4.68\left(\frac{a}{b}\right)^2}$$

$$B = \frac{0.27 - 0.23(ab)}{1 + 2.23\left(\frac{a}{b}\right)}$$

4.4 Mounting Arrangements

The mountings are required to restrain the busbars under all conditions to ensure that:

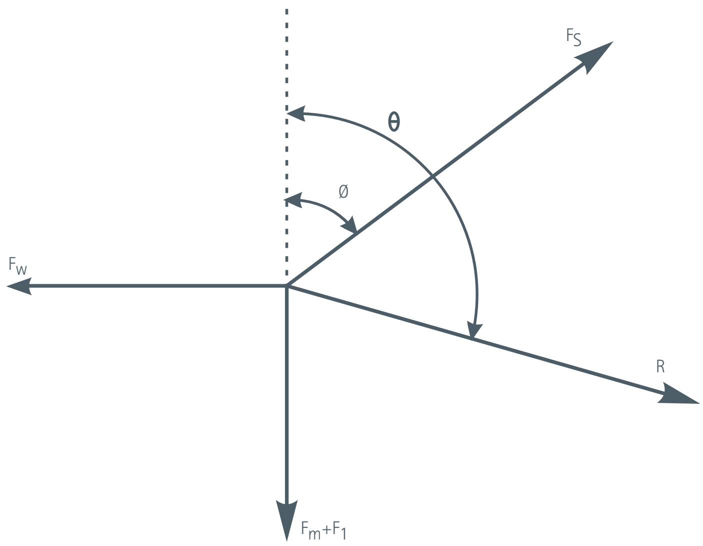
- the material is not over stressed
- a safe distance is maintained between bars in normal service and under short-circuit conditions by limiting maximum deflection
- the bars do not vibrate excessively (which would impair the long term efficacy of joints).

4.4.1 Maximum Permissible Stress

The maximum permissible stress in a conductor is the resultant of:

- its own natural weight (F_m) and
- in the case of busbars installed out of doors, the additional forces of wind (F_W) and ice (F_i) loadings and
- the magnetic forces resulting from a short circuit, noting that the direction of a short-circuit force (F_S) depends on the position of adjacent phases and the direction of the currents in them.

In a general case, the resultant can be calculated by the following method:



$$R = \sqrt{\left[\left(F_s cos\varphi - (F_m + F_i)\right)^2 + (F_s sin\varphi - F_w)^2\right]}$$

and

$$\theta = tan^{-1}\left(\frac{F_m\ sin\varphi - F_w}{F_m\ cos\varphi - (F_m + F_s)}\right)$$

The maximum skin stress in the conductor can then be calculated using the following formula:

$$f = \frac{RDL^2}{16\ I}$$

where:

- f is maximum skin stress, N/mm^2
- L is span, mm
- I is moment of inertia, mm
- D is diameter or depth of rectangular section, mm.

This equation is valid for a uniformly loaded single beam which is freely supported at both ends or freely supported at one end and fixed at the other. For a beam which is horizontally fixed at both ends, maximum skin stress is reduced to one third at the centre and to two-thirds at its ends.

The maximum permissible stress is dependent on the conductor material, temper, etc. but must not exceed the material proof stress otherwise permanent deformation will occur. For a conductor manufactured from hard drawn copper, the value is approximately 245 N/mm^2.

4.4.1.1 Moment of Inertia

In the above formula the moment of inertia, *I*, for the section of the beam has to be calculated about the neutral axis which runs parallel to the beam where the beam has zero tensile forces. In most cases this is the same axis of the centre of cross-section.

Shaped profiles have much higher values of moment of inertia than simple bars and rods (see Section 5.0 Busbar Profiles).

For a rectangular section of depth *D* mm and breadth *B* mm

$$I = \frac{1}{12}BD^3$$

For a circular section of diameter D mm

$$I = \frac{\pi}{64}D^4$$

or

$$I = 0.491D^4$$

For a tubular section of internal diameter *d* mm and external diameter *D* mm

$$I = \frac{\pi}{64}(D^4 - d^4)$$

or

$$I = 0.491\ (D^4 - d^4)$$

It should be noted that the value of *I* for a given cross-section is dependent on the direction in which each individual force is applied.

4.4.2 Deflection

It is important to know how far a bar will deflect so that the necessary clearances between bars and other structures can be maintained.

The maximum deflection of a beam carrying a uniformly distributed load, and freely supported at each end, is given by the formula:

$$\Delta = \frac{5F_m L^4}{384\,EI}$$

where:

Δ is maximum deflection, mm
F_m is weight per unit length of loaded beam, N/mm
L is beam length between supports, mm
E is modulus of elasticity (124 x 10^3N/mm^2)
I is moment of inertia of beam section, mm^4.

If one end of a beam is rigidly fixed in a horizontal position, the deflection is 0.415 times that given by the above formula. If both ends of a beam are rigidly fixed in a horizontal position, the deflection is 0.2 times that given by the above formula.

Thus, with a continuous beam freely supported at several points, the maximum deflection in the centre spans may be assumed to be 0.2 times that given by the formula, while the deflection in the end spans is 0.415 times. Therefore, the deflection in the end spans may be assumed to be twice that in the centre spans, assuming equal span distances.

4.4.3 Natural Frequency

The natural frequency of a beam simply supported at its end is:

$$f_n = \frac{17.74}{\sqrt{\Delta}}$$

and for a beam with both ends fixed horizontally it is:

$$f_n = \frac{18.04}{\sqrt{\Delta}}$$

where:

f_n is natural frequency, Hz
Δ is deflection, mm.

As the deflection with fixed ends is 0.2 times the value with freely supported ends, it follows that the natural frequency is increased by 2.275 times by end-fixing; fixing one end only increases the natural frequency by about 50%.

Busbar systems should be designed to have a natural frequency which is not within 30% of the vibrations induced by the magnetic fields resulting from the currents, including any significant harmonic currents, flowing in adjacent conductors.

Where equipment is to be mounted outside, natural frequencies of less than 2.75 Hz should be avoided to prevent vibration due to wind eddies.